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Nonzero temperature two-mode squeezing for time-dependent two-level systems

J. Aliaga [1], J.L. Gruver [2], A.N. Proto [1], Hilda A. Cerdeira [3]

International Centre for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy

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Abstract

A maximum entropy principle density matrix method, valid for systems with temperature different from zero, is presented making it possible to find two-mode squeezed states in two-level systems with relevant operators and Hamiltonian connected with O(3,2). A method which allows one to relate the appearance of squeezing to the relevant operators, included in order to define the density matrix of the system, is given.

The two-level system is one of the best known and most fruitful models in several fields of physics like quantum optics, magnetic resonance, quantum electronics and chaotic behavior of dynamical systems. Recently, the dynamics and thermodynamics for the two-level system have been studied showing how the dynamical equations can be easily extended to the N-level system problem [1]. In the last ten years considerable attention has been paid to examine the possibility of generating two-mode squeezed states in a pair of coupled oscillators [2–9]. As it is well known the problem of squeezed states may find applications in low-noise optical communications as well as in the gravitational-wave detection due to the fact that squeezed states have manifestly nonclassical properties. Nonzero temperature squeezed states have been recently introduced in systems with relevant operators and Hamiltonians connected with SU(1,1) and isomorphics [10–12].

The aim of this Letter is to describe a maximum entropy principle (MEP) method to find two-mode squeezing, based on a density matrix formalism [10], for the time-dependent two-level systems. Our main results are: (i) to obtain the relevant operators necessary in order to define the density operator of the system, which turns out to be isomorphic to the two-oscillator representation of O(3,2) [13]; (ii) to find the evolution equations for these relevant operators for the time-dependent two-level systems; and (iii) to obtain both zero and nonzero temperature two-mode squeezed states for this quantal system. The MEP formalism is based on the particular form of the density operator $\hat{\rho}$ [10]. From the knowledge of the expectation values of, say, M operators $\hat{O}_j$ ($\hat{O}_0 = \hat{I}$ = identity operator),

$$\langle \hat{O}_j \rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{O}_j], \qquad j = 0, 1, \ldots, M, \tag{1}$$

[1] Permanent address: Grupo de Sistemas Dinámicos, Universidad de Buenos Aires – Centro Regional Norte, C.C. 2, (1638) V. López, Buenos Aires, Argentina.
[2] On leave from Grupo de Sistemas Dinámicos, Universidad de Buenos Aires – Centro Regional Norte, C.C. 2, (1638) V. López, Buenos Aires, Argentina.
[3] Also at Universidade Estadual de Campinas, C.P. 6165, 13081 Campinas, São Paulo, Brazil.

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the density operator is

$$\hat{\rho}(t) = \exp\left(-\lambda_0 \hat{I} - \sum_{j=1}^{M} \lambda_j \hat{O}_j\right), \tag{2}$$

where the $M+1$ Lagrange multipliers λ_j, are determined to fulfill Eq. (1). The density operator $\hat{\rho}$ maximizes the entropy, $S(\hat{\rho})$, given (in units of the Boltzmann constant) by

$$S(\hat{\rho}) = -\mathrm{Tr}[\hat{\rho} \ln \hat{\rho}] = \lambda_0 \hat{I} + \sum_{j=1}^{M} \lambda_j \langle \hat{O}_j \rangle. \tag{3}$$

If we impose that the operator $\hat{\rho}(t)$ obeys the Liouville equation and that the entropy be a constant of motion, it is found that the *relevant* operators entering Eq. (2) are those which close a semi-Lie algebra under commutation with the Hamiltonian $\hat{H}$,

$$[\hat{H}(t), \hat{O}_i] = \mathrm{i}\hbar \sum_{i=0}^{q} g_{ji}(t) \hat{O}_j, \tag{4}$$

where the g_{ji} are the elements (c-numbers) of a $q \times q$ matrix G (which may depend on time if $\hat{H}$ is time dependent). The closure condition (Eq. (4)) on the $\hat{O}_j$ leads to the fact that the time-dependent Schrödinger equation can be replaced by a set of coupled equations for the λ_i,

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}t} = \sum_{j=0}^{q} g_{ij} \lambda_j. \tag{5}$$

The temporal evolution of the expectation values of the operators (Eq. (1)) can be obtained using Ehrenfest's theorem. Assuming that the $\hat{O}_i$ do not depend explicitly on time we find

$$\frac{\mathrm{d}\langle \hat{O}_i \rangle_t}{\mathrm{d}t} = -\sum_{j=0}^{q} g_{ji} \langle \hat{O}_j \rangle_t, \qquad i = 1, 2, \ldots, q. \tag{6}$$

Now, using this formalism we are going to study the main features of the two-mode squeezing for the time-dependent two-level system.

The Hamiltonian which describes the problem reads

$$\hat{H}(t) = E_1 \hat{a}_1^\dagger \hat{a}_1 + E_2 \hat{a}_2^\dagger \hat{a}_2 + T(t) \gamma (\hat{a}_1 \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1^\dagger) \tag{7}$$

($\hbar = 1$), where γ is the interaction energy, $\hat{a}_i^\dagger$ ($\hat{a}_i$) (boson operators) represent the creation (annihilation) of a particle in state i, and $T(t)$ is any function of time. As was said in Ref. [10], in order to properly describe the appearance of squeezing we must know not only the initial mean values of the position and momentum, but also their squares. In doing this, we find two sets of relevant operators (Eq. (4)) associated with the problem. They are

$$\hat{O}_j \equiv \hat{q}_j = \gamma_{j,1} (\hat{a}_j^\dagger + \hat{a}_j), \tag{8}$$

$$\hat{O}_{2+j} \equiv \hat{p}_j = \mathrm{i}\gamma_{j,2} (\hat{a}_j^\dagger - \hat{a}_j), \tag{9}$$

and

$$\hat{O}_{4+j} \equiv \hat{q}_j^2 = \gamma_{j,1}^2 (\hat{a}_j^\dagger \hat{a}_j^\dagger + \hat{a}_j^\dagger \hat{a}_j + \hat{a}_j \hat{a}_j^\dagger + \hat{a}_j \hat{a}_j), \tag{10}$$

$$\hat{O}_{6+j} \equiv \hat{p}_j^2 = -\gamma_{j,2}^2 (\hat{a}_j^\dagger \hat{a}_j^\dagger - \hat{a}_j^\dagger \hat{a}_j - \hat{a}_j \hat{a}_j^\dagger + \hat{a}_j \hat{a}_j), \tag{11}$$

$$\hat{O}_9 \equiv \hat{q}_1\hat{q}_2 = \gamma_{1,1}\gamma_{2,1}(\hat{a}_1^\dagger\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger + \hat{a}_1\hat{a}_2), \tag{12}$$

$$\hat{O}_{10} \equiv \hat{p}_1\hat{p}_2 = -\gamma_{1,2}\gamma_{2,2}(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2 - \hat{a}_1\hat{a}_2^\dagger + \hat{a}_1\hat{a}_2), \tag{13}$$

$$\hat{O}_{10+j} \equiv \tfrac{1}{2}(\hat{q}_j\hat{p}_j + \hat{p}_j\hat{q}_j) = \mathrm{i}\gamma_{j,1}\gamma_{j,2}[(\hat{a}_j^\dagger)^2 - (\hat{a}_j)^2], \tag{14}$$

$$\hat{O}_{13} \equiv \hat{q}_1\hat{p}_2 = i\gamma_{1,1}\gamma_{2,2}(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2), \tag{15}$$

$$\hat{O}_{14} \equiv \hat{p}_1\hat{q}_2 = i\gamma_{1,2}\gamma_{2,1}(\hat{a}_1^\dagger\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2 - \hat{a}_1\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2), \tag{16}$$

where $j = 1,2$ and $\gamma_{j,1} = (1/2E_j)^{1/2}$, $\gamma_{j,2} = (\frac{1}{2}E_j)^{1/2}$. This second set is isomorphic to O(3,2) and is equivalent to Dirac's two-oscillator representation [13]. The structure constants of the semi-Lie algebra can be obtained using Eq. (4). The **g** matrix can be divided in two nonzero blocks, related to the two sets introduced by Eqs. (8), (9) and (10)-(16),

$$\mathbf{g}_1 = \begin{pmatrix} 0 & 0 & -E_1 & -T\gamma \\ 0 & 0 & -T\gamma & -E_2 \\ E_1 & T\gamma & 0 & 0 \\ T\gamma & E_2 & 0 & 0 \end{pmatrix} \tag{17}$$

and

$$\mathbf{g}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -2E_1 & 0 & -2T\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2E_2 & 0 & -2T\gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & 2E_1 & 0 & 0 & 2T\gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2E_2 & 2T\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -T\gamma & -T\gamma & -E_2 & -E_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & T\gamma & T\gamma & E_1 & E_2 \\ E_1 & 0 & -E_1 & 0 & T\gamma & -T\gamma & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & -E_2 & T\gamma & -T\gamma & 0 & 0 & 0 & 0 \\ T\gamma & 0 & 0 & -T\gamma & E_2 & -E_1 & 0 & 0 & 0 & 0 \\ 0 & T\gamma & -T\gamma & 0 & E_1 & -E_2 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

Now, using Eqs. (5), (6) we can write the evolution equations for the mean values of the relevant operators and the Lagrange multipliers. The temporal evolutions of these variables can be easily obtained by solving the sets of first order differential equations (5), (6) for any given function $T(t)$. From this point of view, we can say that *the dynamical problem of this system is completely solved* provided we are able to properly evaluate the initial mean values of the relevant operators.

Thus, in order to study the appearance of squeezing it is necessary to obtain the initial mean values given by Eq. (1). As we are considering a set of noncommuting relevant operators (Eqs. (8)-(16)), we need to diagonalize the logarithm of the density operator $\hat{\rho}$ before applying Eq. (1). This diagonalization is performed by generalizing a method developed in Ref. [10]. We will write the density matrix including the Hamiltonian as a relevant operator in order to derive a thermodynamical approach to the problem at hand. For simplicity we shall consider the system in resonance so that $E_1 = E_2 = E$ [8]. The density operator, in terms of new creation and annihilation operators, reads

$$\hat{\rho}(t) = \exp\left(-\lambda_0\hat{I} - \beta\hat{H} - \sum_{j=1}^{14}\lambda_j\hat{O}_j\right) = \exp\{-\lambda_0\hat{I} + \beta[E(|\gamma_1|^2 + |\gamma_2|^2) + 4T(t)\gamma|\gamma_1||\gamma_2|\cos\psi_1\cos\psi_2]$$
$$- \beta E\omega_1(\hat{d}_1^\dagger\hat{d}_1 + \tfrac{1}{2}) - \beta E\omega_2(\hat{d}_2^\dagger\hat{d}_2 + \tfrac{1}{2})\}, \tag{19}$$

where $\hat{d}_j$, $j = 1, 2$, are new annihilation operators defined as follows,

$$\hat{d}_1 = \frac{1}{2\sqrt{2}}[\omega_1^+(\hat{b}_1 - \hat{b}_2) + \omega_1^-(\hat{b}_1^\dagger - \hat{b}_2^\dagger)], \tag{20}$$

$$\hat{d}_2 = \frac{1}{2\sqrt{2}}[\omega_2^+(\hat{b}_1 + \hat{b}_2) + \omega_2^-(\hat{b}_1^\dagger + \hat{b}_2^\dagger)t], \tag{21}$$

$$\omega_j^+ = \omega_j^{1/2} + \omega_j^{-1/2}, \qquad \omega_j^- = \omega_j^{1/2} - \omega_j^{-1/2}, \tag{22}$$

$$\omega_1^2 = 1 - \frac{2T(t)\gamma}{E}, \qquad \omega_2^2 = 1 + \frac{2T(t)\gamma}{E}, \tag{23}$$

so that the system can be decoupled [9], and the $\hat{b}_j^\dagger$, $j = 1, 2$, are the two mode version of the transformation introduced in Ref. [10],

$$\hat{b}_j^\dagger = |\cosh r_j| \exp(\mathrm{i}\varphi_j)\hat{a}_j^\dagger + |\sinh r_j| \exp(-\mathrm{i}\theta_j)\hat{a}_j + |\gamma_j| \exp(-\mathrm{i}\psi_j). \tag{24}$$

Eqs. (20), (21) are valid if and only if $\omega_j^2 \geqslant 0$. *This condition introduces an upper bound to the value of the interaction* (i.e., $2T(t)\gamma \leqslant E$). This relation has been previously obtained in Ref. [14], and is necessary in order to deal with a properly defined density operator (i.e., normalizable and positive definite). The diagonalization of the density operator (Eq. (19)) introduces some restrictions in the Lagrange multipliers dual space, which are the consequence of the quantum statistical character of the system under study [10]. It can be seen that the Lagrange multipliers $\{\lambda_5, \ldots, \lambda_{14}\}$ can be written in terms of six parameters, $\{r_1, \theta_1, \varphi_1, r_2, \theta_2, \varphi_2\}$, introduced by Eq. (24). They read

$$\frac{4\tilde{\lambda}_5'}{E} = (C_1^+)^2 + (S_1^-)^2 - 1, \qquad \frac{4\tilde{\lambda}_6'}{E} = (C_2^+)^2 + (S_2^-)^2 - 1, \qquad \frac{4\tilde{\lambda}_7'}{E} = (S_1^+)^2 + (C_1^-)^2 - 1,$$

$$\frac{4\tilde{\lambda}_8'}{E} = (S_2^+)^2 + (C_2^-)^2 - 1, \qquad \frac{\tilde{\lambda}_9'}{T\gamma} + \frac{1}{2} = C_2^+ C_1^+, \qquad \frac{\tilde{\lambda}_{10}'}{T\gamma} + \frac{1}{2} = S_2^+ S_1^+,$$

$$\frac{2\tilde{\lambda}_{11}'}{E} = S_1^+ C_1^+ + S_1^- C_1^-, \qquad \frac{2\tilde{\lambda}_{12}'}{E} = S_2^+ C_2^+ + S_2^- C_2^-, \qquad \frac{\tilde{\lambda}_{13}'}{T\gamma} = S_2^+ C_1^+, \qquad \frac{\tilde{\lambda}_{14}'}{T\gamma} = C_2^+ S_1^+, \tag{25}$$

where $S_j^\pm = |\sinh r_j| \sin\theta_j \pm |\cosh r_j| \sin\varphi_j$, $C_j^\pm = |\cosh r_j| \cos\varphi_j \pm |\sinh r_j| \cos\theta_j$, $j = 1, 2$, and the $\tilde{\lambda}_i'$ are dimensionless Lagrange multipliers (i.e., $\tilde{\lambda}_5 \equiv \gamma_{1,1}^2 \lambda_5$) divided by β, which play the role of *intensive variables* in a quantum thermodynamical formalism [10]. The functions $S_j^\pm$ and $C_j^\pm$ verify a relation similar to that of the hyperbolic functions,

$$C_j^+ C_j^- - S_j^+ S_j^- = 1, \qquad j = 1, 2. \tag{26}$$

The mean values of the relevant operators can be evaluated using Eq. (1), where the operators and the density matrix must be written in terms of $\hat{d}_j$ and $\hat{d}_j^\dagger$, $j = 1, 2$. The main results are

$$\Delta\hat{q}_j^2 = \tfrac{1}{2}\gamma_{j,1}^2 \sum_{i=1}^{2} \left(\omega_i (S_j^+)^2 + \frac{(C_j^-)^2}{\omega_i}\right) \coth(\tfrac{1}{2}\beta\omega_i), \tag{27}$$

$$\Delta\hat{p}_j^2 = \tfrac{1}{2}\gamma_{j,2}^2 \sum_{i=1}^{2} \left(\omega_i (C_j^+)^2 + \frac{(S_j^-)^2}{\omega_i}\right) \coth(\tfrac{1}{2}\beta\omega_i), \tag{28}$$

$$\Delta\hat{q}_1\hat{q}_2 = \tfrac{1}{2}\gamma_{1,1}\gamma_{2,1} \sum_{i=1}^{2} (-1)^i \left(\omega_i S_1^+ S_2^+ + \frac{C_2^- C_1^-}{\omega_i}\right) \coth(\tfrac{1}{2}\beta\omega_i), \tag{29}$$

$$\Delta\hat{p}_1\hat{p}_2 = \tfrac{1}{2}\gamma_{1,2}\gamma_{2,2}\sum_{i=1}^{2}(-1)^i\left(\omega_i C_1^+ C_2^+ + \frac{S_2^- S_1^-}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i), \tag{30}$$

$$\Delta\hat{q}_j\hat{p}_j = -\tfrac{1}{2}\gamma_{j,1}\gamma_{j,2}\sum_{i=1}^{2}\left(\omega_i S_j^+ C_j^+ + \frac{S_j^- C_j^-}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i), \tag{31}$$

$$\Delta\hat{q}_1\hat{p}_2 = \tfrac{1}{2}\gamma_{1,1}\gamma_{2,2}\sum_{i=1}^{2}(-1)^{i+1}\left(\omega_i S_1^+ C_2^+ + \frac{S_2^- C_1^-}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i), \tag{32}$$

$$\Delta\hat{q}_2\hat{p}_1 = \tfrac{1}{2}\gamma_{2,1}\gamma_{1,2}2\sum_{i=1}^{2}(-1)^{i+1}\left(\omega_i S_2^+ C_1^+ + \frac{S_1^- C_2^-}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i), \tag{33}$$

$j = 1,2$, where $\Delta\hat{O}_i\hat{O}_j \equiv \langle\hat{O}_i\hat{O}_j\rangle - \langle\hat{O}_i\rangle\langle\hat{O}_j\rangle$. Eqs. (27), (28) are the dispersions of $\hat{q}_j$ and $\hat{p}_j$, respectively. From these equations it can be seen that the system will be squeezed in $\hat{q}_j$ or $\hat{p}_j$ when

$$\frac{1}{2}\sum_{i=1}^{2}\left(\omega_i (S_j^+)^2 + \frac{(C_j^-)^2}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i) < 1, \tag{34}$$

$$\frac{1}{2}\sum_{i=1}^{2}\left(\omega_i (C_j^+)^2 + \frac{(S_j^-)^2}{\omega_i}\right)\coth(\tfrac{1}{2}\beta\omega_i) < 1, \tag{35}$$

respectively. So, Eqs. (34), (35) define the conditions for the appearance of *nonzero temperature two mode squeezed states*. For the zero temperature case $\coth(\frac{1}{2}\beta\omega_i) = 1$. This states, defined via Eq. (19), are some kind of nonzero temperature generalization of the two-mode Gaussian pure states described in Ref. [4]. If one introduces the two-dimensional covariance matrices $\mathbf{S}_{\tilde{q}}$, $\mathbf{S}_{\tilde{p}}$, and $\mathbf{S}_{\tilde{q}\tilde{p}}$,

$$\mathbf{S}_{\tilde{q}} = \begin{pmatrix} \Delta\tilde{\hat{q}}_1^2 & \Delta\tilde{\hat{q}}_1\tilde{\hat{q}}_2 \\ \Delta\tilde{\hat{q}}_1\tilde{\hat{q}}_2 & \Delta\tilde{\hat{q}}_2^2 \end{pmatrix}, \tag{36}$$

$$\mathbf{S}_{\tilde{p}} = \begin{pmatrix} \Delta\tilde{\hat{p}}_1^2 & \Delta\tilde{\hat{p}}_1\tilde{\hat{p}}_2 \\ \Delta\tilde{\hat{p}}_1\tilde{\hat{p}}_2 & \Delta\tilde{\hat{p}}_2^2 \end{pmatrix}, \tag{37}$$

$$\mathbf{S}_{\tilde{q}\tilde{p}} = \begin{pmatrix} \Delta\tilde{\hat{q}}_1\tilde{\hat{p}}_1 & \Delta\tilde{\hat{q}}_1\tilde{\hat{p}}_2 \\ \Delta\tilde{\hat{p}}_1\tilde{\hat{q}}_2 & \Delta\tilde{\hat{q}}_2\tilde{\hat{p}}_2 \end{pmatrix}, \tag{38}$$

where $\tilde{\hat{O}}_j$ are dimensionless operators (i.e., $\tilde{\hat{O}}_5 \equiv \hat{O}_5/\gamma_{1,1}^2$, it can be shown that $\mathbf{S}_{\tilde{q}}\mathbf{S}_{\tilde{p}} - \mathbf{S}_{\tilde{q}\tilde{p}}^2 = \frac{1}{2}[\coth^2(\frac{1}{2}\beta\omega_1) + \coth^2(\frac{1}{2}\beta\omega_2)]\mathbf{I}$. For zero temperature (i.e., $\beta = \infty$) this relation is equal to Eq. (3.2.11a) of Ref. [4]. Therefore, our MEP density matrix in the zero-temperature limit has the same property that the two-mode Gaussian pure states. The case with $\theta_1 = 0$, $\theta_2 = 0$, $\varphi_1 = 0$, and $\varphi_2 = 0$ has the remarkable property that $S_j^{\pm} = 0$, $j = 1,2$. From Eqs. (32)–(34) it can be seen that for this special initial condition $\mathbf{S}_{\hat{q}\hat{p}} = 0$ (i.e., they are the states with minimum uncertainty product). It must be noticed that the states obtained in our work are different from the ones obtained in Ref. [4] because our density operator evolves following the Liouville equation (i.e., our states depend on the Hamiltonian of the system we are trying to describe). This can be seen from the fact that the mean values of the relevant operators and the Lagrange multipliers depend on ω_j, $j = 1,2$. We want to stress the fact that all these results are valid for any given temperature (i.e., mixture state) and for any $T(t)$.

Now, we will show a particular case as an example. Let us consider the time-independent interaction $T(t) = 1$. For this special case we obtain that $\hat{O}_5 + \hat{O}_6 + \hat{O}_7 + \hat{O}_8$ and $\hat{O}_9 + \hat{O}_{10}$ are constants of motion. The temporal

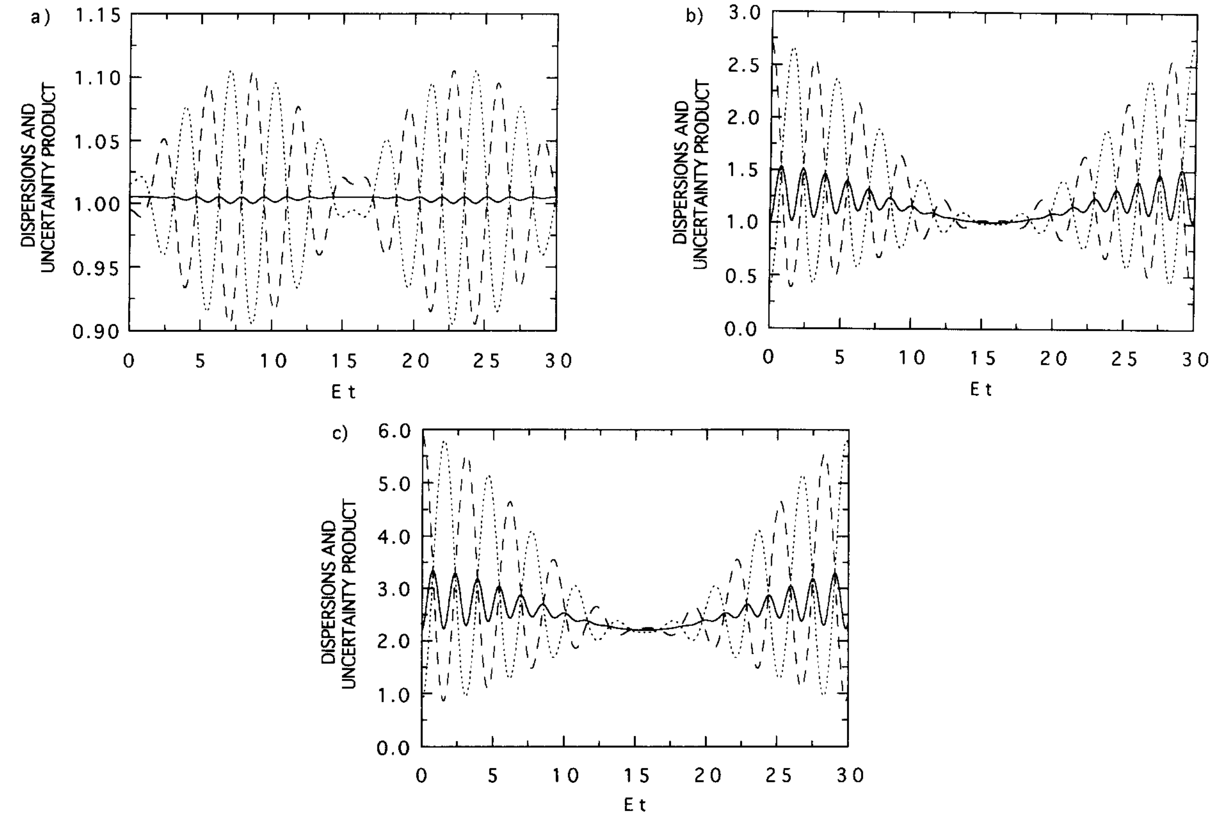

Fig. 1. Temporal evolution of the dispersions $\Delta\tilde{q}_1^2$ (dotted line), $\Delta\tilde{p}_1^2$ (dashed line), and dimensionless uncertainty product (full line): (a) $\beta = \infty$, $r_1 = 0$; (b) $\beta = \infty$, $r_1 = 0.5$; (c) $\beta = 1/E$, $r_1 = 0.5$. For the three cases $\gamma/E = 0.1$, $\theta_1 = 0$, $\varphi_1 = 0$, $r_2 = 0$, $\theta_2 = 0$, $\varphi_2 = 0$.

behavior of the mean values of the relevant operators and the Lagrange multipliers are linear combinations of sines and cosines with frequencies $2E$, 2γ, $2(E+\gamma)$, and $2(E-\gamma)$. The detailed solution, in terms of the initial mean values of the relevant operators will be published elsewhere. Some particular examples of the temporal evolution of the dispersions $\Delta\tilde{q}_1^2$, $\Delta\tilde{p}_1^2$ and dimensionless uncertainty product are shown in Fig. 1. We considered three different cases: (a) $\beta = \infty$, $r_1 = 0$, (b) $\beta = \infty$, $r_1 = 0.5$, (c) $\beta = 1/E$, $r_1 = 0.5$, and for the three cases $\gamma/E = 0.1$, $\theta_1 = 0$, $\varphi_1 = 0$, $r_2 = 0$, $\theta_2 = 0$, and $\varphi_2 = 0$. For zero temperature (i.e., $\beta = \infty$, cases (a), (b)) the system has squeezing notwithstanding the values of the parameters. When r_1 and (or) r_2 are small (large) the squeezing is small (large) but the amount of time the system is squeezed compared with the period is large (small). For nonzero temperature (case (c)) the system has squeezing only if r_1 or r_2 are large enough so as to fulfill Eqs. (34), (35).

Summarizing, we have presented a formalism which allows to study the appearance of squeezing for time-dependent two-level systems. We obtained the relevant operators related with the problem at hand and found their dynamical evolution equations. It was possible to diagonalize the density operator and evaluate the initial mean values of these relevant operators. This allows us to analyze squeezing in terms of six parameters, letting us work with a properly defined quantum statistical density operator. Finally, the time independent case was considered, showing the possibility of obtaining two-mode squeezed states for zero and nonzero temperature.

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