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Turbulence, chaos and noise in globally coupled Josephson junction arrays

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Abstract

We discuss the effects of both temporal and quenched noise in underdamped Josephson junction series arrays that are globally coupled through a resistive load and driven by an rf current. We study the breakdown of the law of large numbers in the turbulent phase of the Josephson arrays. This corresponds to a saturation of the broad band noise S_0 for a large number N of junctions. We find that this phenomenon is stable against temporal noise (thermal fluctuations) and quenched noise (disorder). The behavior of S_0 versus temperature T, for large N, shows three different regimes. For $0 < T < T_{c1}$, S_0 decreases when increasing T, and there is turbulence and the breakdown of the law of large numbers. For $T_{c1} < T < T_{c2}$, S_0 is constant and the dynamics is dominated by the chaos of the individual junctions. Finally for $T > T_{c2}$, S_0 is mainly due to thermal fluctuations, since it increases linearly with T.

1. Introduction

Josephson junction arrays are mesoscopic devices which can be fabricated with very specific properties (see, e.g., Ref. [1]). In the last years they have become a good laboratory for the study of nonlinear dynamical systems with many degrees of freedom [2-8]. Moreover, they have potential applications as high frequency coherent power sources [9,10], parametric amplifiers and voltage standards [9]. One of the prototype models of nonlinear systems with many degrees of freedom are coupled logistic maps [11]. In particular, globally coupled maps (GCM) have been studied as a mean field type extension of these models [12]. As a consequence of the interplay between temporal chaos and space synchronization, the GCM exhibit coherent, ordered, partially ordered and turbulent phases [12]. In the turbulent phase, even when the spatial coherence is completely destroyed, a subtle collective behavior emerges. This was seen as a violation of the law of large numbers [13-17] when increasing the number of logistic maps.

Recently, we have studied a physical realization of the GCM in one-dimensional Josephson junction series arrays (JJSA) [7,8]. In this system, the role of the logistic maps is played by underdamped single Josephson junctions, which can have chaotic dynamics when driven by an rf bias current [18,19]. The global coupling is achieved by connecting the junctions in series but with a common resistive shunting load. Therefore, the two conflicting trends of GCM are present: destruction of coherence due to the chaotic divergences of the individual junctions, and synchronization through the global averaging of the common shunting load.

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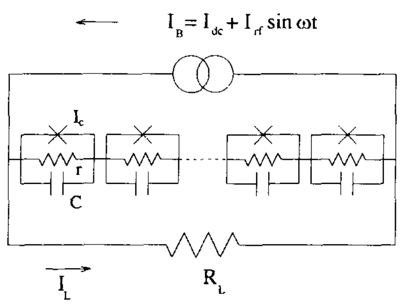

Fig. 1. Schematic circuit of a Josephson junction series array with a resistive load R_L and external current bias I_B. Each Josepshon junction, with critical current I_c, is modeled including a shunt resistance r and a capacitance C.

We have found that the breakdown of the law of large numbers can be observed in rf-driven underdamped JJSA, accompanied in this case by an emergence of novel pseudo Shapiro steps [7]. We have also studied the different regimes in the spatio-temporal dynamics of the JJSA, finding coherent, ordered, partially ordered, turbulent and quasiperiodic phases, depending on the dc component of the bias current [7,8].

Our previous studies have been performed neglecting thermal fluctuations and disorder in the JJSA. In this Letter we will consider the effects of a finite temperature and the effects of the disorder on the turbulent phase of JJSA, for two reasons. (i) The thermal noise cannot be ignored if we want to encourage real experiments in this system (then we must know if the breakdown of the law of large numbers is stable at finite temperatures). Also, in the real JJSA the junctions are not exactly identical: the values of the critical currents of the junctions have typically a spread of 5% or 1% in the best cases [9]. (ii) The addition of noise in the dynamics of GCM has shown interesting effects in previous studies [13,14].

2. Dynamics of Josephson junction series arrays: turbulent phase

Let us consider an array of underdamped Josephson junctions connected in series, shunted by a resistive load in parallel [2,3], and subjected to an *rf bias current* $I_B(t) = I_{dc} + I_{rf}\sin(\omega_{rf}t)$. A schematic representation of this circuit is shown in Fig. 1.

The dynamical behavior of the junction k in the array is given by

$$I_S = I_{c,k}\sin\phi_k + \frac{\hbar}{2er}\frac{d\phi_k}{dt} + \frac{C\hbar}{2e}\frac{d^2\phi_k}{dt^2} + \Gamma_k(t),\quad k = 1,\dots,N, \tag{1}$$

where ϕ_k is the superconducting phase difference in the Josephson junction k, $I_{c,k}$ is its corresponding critical current, r is the quasiparticle resistance of the junctions, C is the capacitance of the junctions and I_S is the current flowing through the circuit branch with the junctions in series. The Johnson noise term $\Gamma_k(t)$ satisfies $\langle\Gamma_k(t)\Gamma_{k'}(t')\rangle = (2kT/r)\delta_{k,k'}\delta(t-t')$, with T the temperature. Eqs. (1) correspond to the resistively shunted junction model [20], commonly used to describe the behavior of current biased Josephson junctions [21].

On the other hand, the common resistive load satisfies,

$$I_L = \frac{1}{R_L}\sum_{k=1}^{N} V_k + \Gamma_L(t) = \sum_{k=1}^{N}\frac{\hbar}{2eR_L}\frac{d\phi_k}{dt} + \Gamma_L(t), \tag{2}$$

where R_L is the resistance of the load, I_L is the current flowing through the load, $V_k = (\hbar/2e)(d\phi_k/dt)$ is the voltage drop in the junction k, and $\Gamma_L(t)$ is the Johnson noise in the shunting load ($\langle\Gamma_L(t)\Gamma_L(t')\rangle = (2kT/R_L)\delta(t-t')$). The external bias current divides between the load and the junctions in series,

$$I_B(t) = I_{dc} + I_{rf}\sin(\omega_{rf}t) = I_S + I_L. \tag{3}$$

Therefore, the governing equations of the JJSA in reduced units are

$$\ddot\phi_k + g\dot\phi_k + i_{c,k}\sin\phi_k + (2\tilde{T}g)^{1/2}\eta_k(\tau) + \frac{\sigma}{N}\sum_{j=1}^{N} g\dot\phi_j + \left(\frac{2\tilde{T}g\sigma}{N}\right)^{1/2}\eta_L(\tau) = i_{dc} + i_{rf}\sin(\Omega_{rf}\tau). \tag{4}$$

We have used the following normalizations: currents are normalized by the nominal critical current $I_c = \langle I_{c,k}\rangle$, $i = I/I_c$; the time is normalized by the plasma frequency $\omega_p t = \tau$, with $\omega_p = \sqrt{2eI_c/\hbar C}$, and voltages are normalized by rI_c. The normalized rf frequency is $\Omega_{rf} = \omega_{rf}/\omega_p$. The thermal Johnson noise is given by the white noise terms $\eta(\tau)$, such that $\langle\eta_k(\tau)\rangle = 0$, $\langle\eta_k(\tau)\eta_{k'}(\tau')\rangle = \delta(\tau-\tau')\delta_{k,k'}$. Temperature is normalized as $\tilde{T} = 2ekT/\hbar I_c$. The parameters in the equations are $g = (\hbar/2eCr^2I_c)^{1/2}$ and $\sigma =$

rN/R_L. Here σ represents the strength of the global coupling in the array. Note that when $\sigma = 0$ Eq. (4) reduces to a set of N independent junctions. The voltage per junction,

$$v(t) = \frac{1}{N}\sum_j v_j = \frac{1}{N}\sum_j g\dot{\phi}_j,$$

acts as a mean field variable. We consider $i_{c,k} = 1+\delta\gamma_k$ with γ_k a random independent noise with normal distribution, and δ the disorder strength. We integrate numerically Eqs. (4) with a second order Runge–Kutta method suitable for stochastic differential equations [22], with step $\Delta\tau = T/160$ with $T = 2\pi/\Omega_{rf}$, for integration times $t = 1024T$, after discarding the first 256 periods. For each run we used different sets of random initial conditions $\{\phi_k(0), \dot{\phi}_k(0)\}$.

Let us first review the case without thermal fluctuations, $\tilde{T} = 0$, and without disorder, $\delta = 0$. The simplest attractor of the system is the coherent state for which $\phi_k(\tau) = \phi_j(\tau) = \phi_0(\tau)$. The equations reduce to the single junction dynamics,

$$\ddot{\phi}_0 + \tilde{g}\dot{\phi}_0 + \sin\phi_0 = i_{\text{bias}}(\tau), \quad (5)$$

with $\tilde{g} = g(1+\sigma)$. It is known that the single Josephson junction can have chaotic behavior in the underdamped regime (for $\tilde{g} < 2$) below the plasma frequency ($\Omega_{rf} < 1$) [19].

One of the responses that can be measured experimentally are the I–V characteristics of the JJSA, which is the time average voltage per junction,

$$v = \frac{1}{N}\sum_j \langle v_j(t)\rangle = \frac{1}{N}\sum_j g\langle\dot{\phi}_j(t)\rangle,$$

as a function of i_{dc}. When the junctions are rf-biased, they can show Shapiro steps [23,19]. These are regions for which the average voltage is constant and given by $v = (n/m)g\Omega_{rf}$. They correspond to phase locked states, which are periodic solutions in resonance with the rf current, either harmonic ($m = 1$), or subharmonic ($m > 1$). In other parts of the I–V characteristic it is possible to have chaotic solutions, in which the junction switches pseudorandomly between unstable, overlapping Shapiro steps [18,19]. We study the chaotic nature of the solutions by computing the maximum Lyapunov exponent λ of the JJSA. Experimentally, most chaotic modes can be observed as broad band noise in the power spectrum of the voltage [18,19]. The power spectrum is computed as

$$S(\omega) = \frac{2}{T_m}\left|\int_0^{T_m} v(\tau)\,e^{i\omega\tau}\,d\tau\right|^2. \quad (6)$$

In the presence of broad band noise, the low frequency part of the spectrum approaches a constant, $S_0 = \lim_{\omega\to 0} S(\omega)$.

We study the spatial behavior of the JJSA through the concept of "clustering" [12]. After the system has fallen in an attractor, we say that two junctions i, j belong to the same cluster if $\phi_i(t) = \phi_j(t) + 2\pi n$ with n an integer. An attractor can be characterized by the number of clusters it has, n_{cl}, and the number of elements of each cluster ($M_1, M_2, \ldots, M_{n_{cl}}$). For example, the coherent state is a one-cluster attractor ($n_{cl} = 1, M_1 = N$). Using these tools, we have studied the I–V characteristics of the JJSA for $\tilde{T} = 0$, calculating λ, S_0 and n_{cl} as a function of the bias current i_{dc} [7,8]. We found that there is (i) an ordered regime, which is periodic in time (it corresponds to Shapiro steps in the I–V characteristics), and is ordered in space in a finite number of "clusters" with the same phase, (ii) a coherent regime, with all the phases equal, (iii) a partially ordered regime, and (iv) a turbulent regime, where there is chaos both in time and space (all the junction phases are different at a given time). In Fig. 2 we show the I–V characteristics, the Lyapunov exponent and the number of clusters n_{cl} for a JJSA with $N = 128$ junctions, coupling $\sigma = 0.4$, and parameters $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, and $i_{rf} = 0.61$. We mainly show here the range of i_{dc} where there is a turbulent phase, characterized by $\lambda > 0$ and $n_{cl} \approx N$. This is the regime that shows the most notable changes when increasing the number of junctions N [7,8].

First of all, let us note that the voltage per junction,

$$v^{(N)}(t) = \frac{1}{N}\sum_{j=1}^{N} g\dot{\phi}_j,$$

acts as a "mean field" in Eq. (4). Since in the turbulent phase the $\phi_j(t)$ take random values almost independently, one might expect that $v(t)$ will behave as an average noise. The power spectrum of $v(t)$ will be

$$S(\omega) = \frac{1}{N}|v_j(\omega)|^2 + \frac{1}{N^2}\left(\sum_{i\neq j} v_i(\omega)v_j^*(\omega)\right), \quad (7)$$

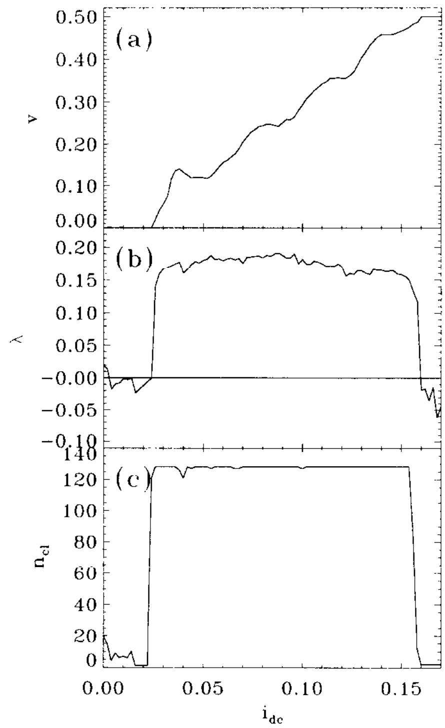

Fig. 2. Behavior of a Josephson junction series array as a function of i_{dc}. For coupling $\sigma = 0.4$, $N = 128$ junctions and parameters $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, $i_{rf} = 0.61$. (a) Average voltage v (I–V characteristics). (b) Maximum Lyapunov exponent λ. (c) Number of clusters n_{cl}.

with $v_j(\omega)$ the Fourier transform of $v_j(t) = g\dot{\phi}_j(t)$. If the $\phi_j(t)$ are completely independent, the second term will vanish for low frequencies, $\omega \to 0$. Therefore $S_0^{(N)} \sim (1/N)S_0^{(1)}$, with $S_0^{(N)}$ the low frequency part of the power spectrum of a JJSA with N junctions. This is the equivalent of the law of large numbers for a periodically driven system. However, we have found that within the turbulent phase S_0 saturates for large N, evidencing a breakdown of the law of large numbers [7], as observed in GCM [13–16]. This is shown in Fig. 3 for given values of σ and i_{dc}. At the same time some pseudo-steps emerge in the I–V characteristics for large N. The phenomenon of pseudo-steps has been discussed by us in Refs. [7,8]. They are evidenced in Fig. 2a within the turbulent phase.

The breakdown of the law of large numbers in GCM has been interpreted by Kaneko [13] as a hidden coherence in the turbulent regime. This coherence shows, for example, an emergence of broad peaks in the power

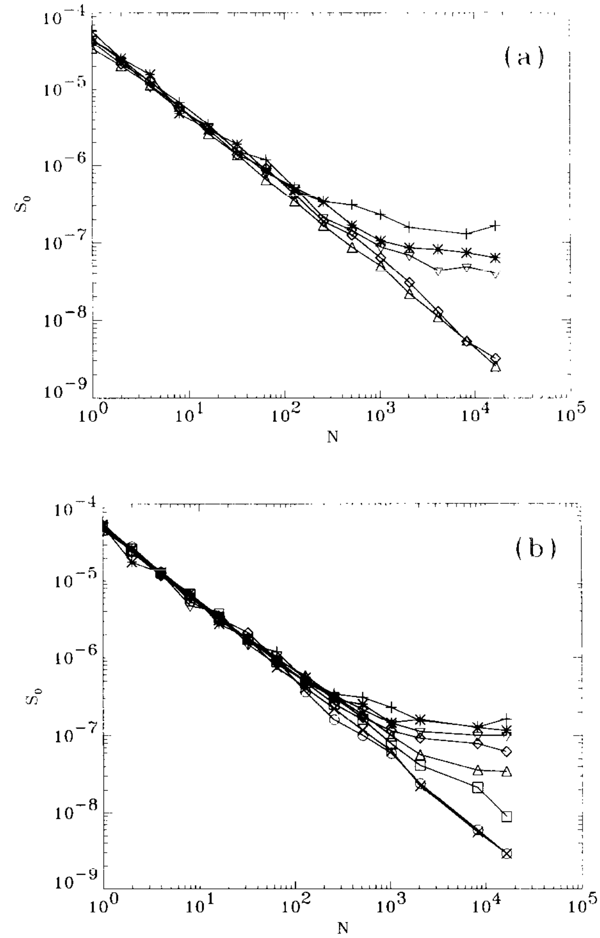

Fig. 3. Low frequency limit of the power spectrum, $S_0 = \lim_{\omega\to 0} S(\omega)$, as a function of the size of the array N. $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, $i_{rf} = 0.61$, $i_{dc} = 0.124$, $\sigma = 0.4$. (a) For varying disorder and zero temperature: (+) $\delta = 0$, (∗) $\delta = 0.005$, (▽) $\delta = 0.01$, (◇) $\delta = 0.02$, (△) $\delta = 0.05$. (b) For different temperatures and no disorder ($\delta = 0$): (+) $\tilde{T} = 0$, (∗) $\tilde{T} = 1 \times 10^{-6}$, (▽) $\tilde{T} = 2 \times 10^{-6}$, (◇) $\tilde{T} = 5 \times 10^{-6}$, (△) $\tilde{T} = 1 \times 10^{-5}$, (□) $\tilde{T} = 2 \times 10^{-5}$, (×) $\tilde{T} = 5 \times 10^{-5}$, (○) $\tilde{T} = 1 \times 10^{-4}$.

spectrum of the mean field variable [13,14]. However, an understanding of the origin of this hidden coherence and the frequency dependence of these broad peaks is still lacking in this problem [13–15,17]. We have also found an emergence of broad peaks in the power spectrum of $v(t)$ for large N coexisting with the breakdown of the law of large numbers [8].

3. Effects of noise: quenched disorder and thermal fluctuations

Let us study the effect of both temporal and quenched noise on the breakdown of the law of large numbers. Particularly, we focus in this paper in its experimental consequences in measurable variables in JJSA.

First, we discuss the effect of quenched disorder at zero temperature, $\tilde{T} = 0$. In Fig. 3a, we show S_0 as a function of N for $\sigma = 0.4$ and $i_{dc} = 0.124$ (which corresponds to the turbulent regime) for increasing disorder $\delta = 0.0, 0.005, 0.01, 0.02, 0.05$. We see that the breakdown of the law of large numbers is stable below a critical disorder $\delta_c \approx 0.015$. For $\delta > \delta_c$ we recover the $1/N$ law for the broad band noise. Therefore, the effects we described in Refs. [7,8] can only be observed in arrays with a 1% spread in the critical currents at most.

In Fig. 3b we show S_0 as a function of N for the same σ and i_{dc} as before, without disorder ($\delta = 0$) and with different values of the temperature. We see that for $\tilde{T} = 0$, S_0 saturates for large N. This breakdown of the law of large numbers is stable for small temperatures, and only after a critical $\tilde{T}_{c1} \approx 4 \times 10^{-5}$ there is a crossover to a $1/N$ behavior. A similar phenomenon has been found when adding a white noise term to GCM [13], where also the $1/N$ behavior is recovered after a critical value of noise intensity.

More interesting, from the experimental point of view, is the behavior of S_0 as a function of temperature for a large number of junctions (above saturation for $\tilde{T} = 0$). In Fig. 4 we show the results for bias $i_{dc} = 0.124$, $\sigma = 0.4$ and $N = 16384$ junctions. We find three different thermal regimes.

(i) For $\tilde{T} < \tilde{T}_{c1} \approx 4 \times 10^{-5}$, the broad band noise *decreases* with increasing temperature. This counter-intuitive behavior is a consequence of the fact that there is a breakdown of the law of large numbers at $\tilde{T} = 0$. The addition of thermal noise reduces in part the subtle coherence that made S_0 saturate for large N. In other words, the typical $N = N_*$ for saturation of S_0 increaseas with increasing noise. This leads to a decrease of S_0 when increasing $\tilde{T}$ for a fixed N. Since there is still a breakdown of the law of large numbers, this is the temperature regime where the *turbulence* and the global coupling of the JJSA are manifested.

(ii) For $\tilde{T}_{c1} < \tilde{T} < \tilde{T}_{c2}$, with $\tilde{T}_{c2} \approx 5 \times 10^{-3}$, S_0

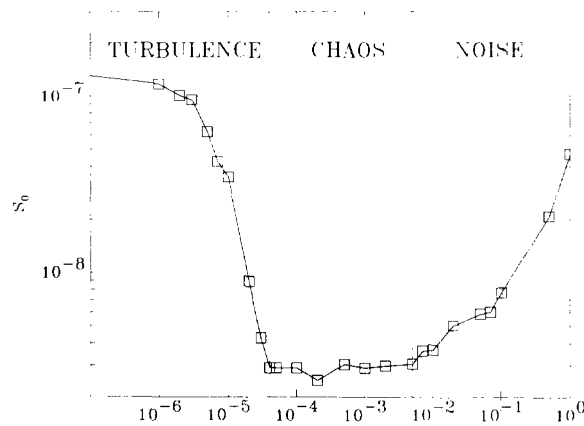

Fig. 4. Low frequency limit of the power spectrum, $S_0 = \lim_{\omega \to 0} S(\omega)$, as a function of the temperature $\tilde{T}$ for a large array, $N = 16384$. $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, $i_{rf} = 0.61$, $i_{dc} = 0.124$, $\sigma = 0.4$.

remains *constant*. Now the $1/N$ law is fulfilled. Here the ϕ_j act as independent chaotic variables. In this temperature regime, the subtle coherence of the global coupling has been destroyed, and S_0 is basically due to the *chaos* of the individual junctions.

(iii) For $\tilde{T} > \tilde{T}_{c2}$, S_0 *increases* with temperature. Here the dynamics of the junctions is dominated by the thermal fluctuations, and therefore the broad band noise S_0 is a consequence of the *thermal noise*.

The thermal fluctuations affect the full power spectrum of $v(t)$ in a surprising way. Perez et al. [14] found that in GCM the broad peaks in the power spectrum become sharper when increasing the noise. In Fig. 5 we show the power spectrum for $\sigma = 0.4$, $i_{dc} = 0.124$ for different temperatures and $N = 16384$. In the absence of thermal fluctuations, $\tilde{T} = 0$, we see in Fig. 5a that there are broad peaks in the power spectrum for frequencies $\omega < \omega_{rf}$. As stated before, these peaks are a consequence of the hidden correlations existing in the turbulent phase [13], due to the breakdown of the law of large numbers. Here we see that when adding a finite temperature, the broad peaks get sharper an better defined when increasing $\tilde{T}$ (Figs. 5b–5d). Only after $\tilde{T} > \tilde{T}_{c1}$ the power spectrum starts to become broadened by the thermal fluctuations (Figs. 5e, 5f).

More quantitatively, following Ref. [14], we define a measure of sharpness,

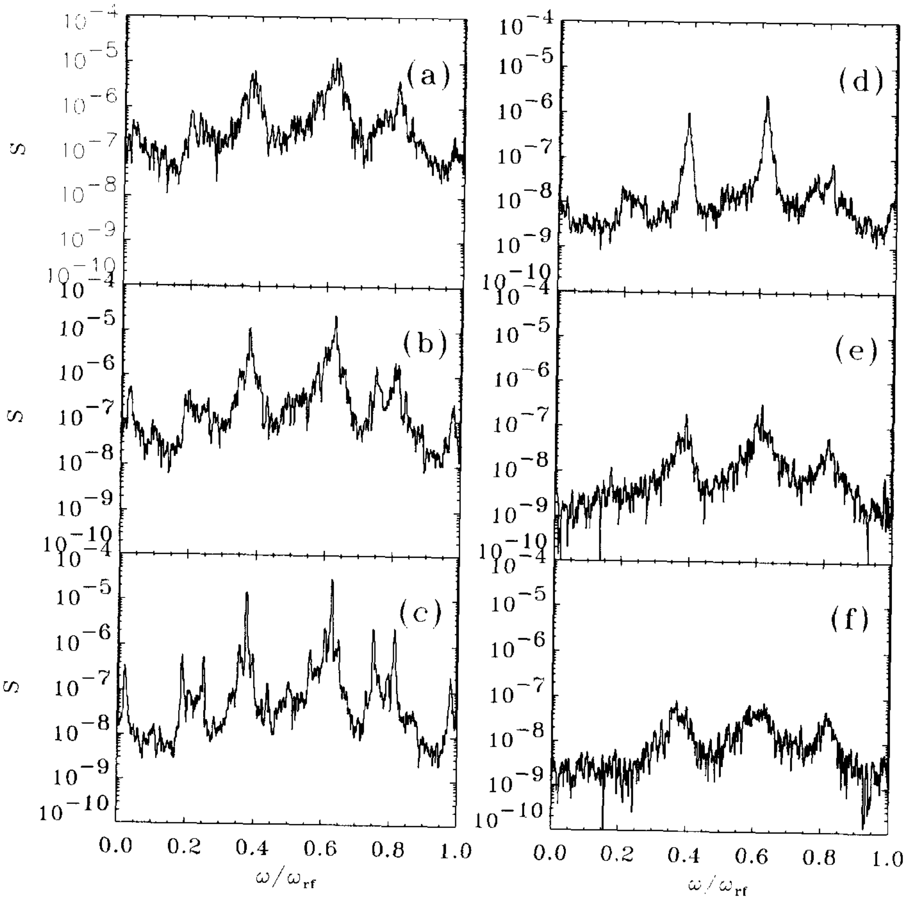

Fig. 5. Power spectrum of the voltage for different temperatures. $\tilde{g} = 0.2$, $\Omega_{\rm rf} = 0.8$, $i_{\rm rf} = 0.61$, $i_{\rm dc} = 0.124$, $\sigma = 0.4$, $N = 16384$. (a) $\tilde{T} = 0$; (b) $\tilde{T} = 2 \times 10^{-6}$; (c) $\tilde{T} = 1 \times 10^{-5}$; (d) $\tilde{T} = 3 \times 10^{-5}$; (e) $\tilde{T} = 2 \times 10^{-4}$; (f) $\tilde{T} = 5 \times 10^{-4}$.

$$W = -\log_{10}\left(\frac{1}{M}\frac{\sum_{l=1}^{M}\sum_{m=1}^{M} S(\omega_{l+m})S(\omega_m)}{\sum_{l=1}^{M} S(\omega_l)^2}\right), \tag{8}$$

where M is the number of discrete points in the spectrum. For a completely flat spectrum $W = 0$, and for a set of δ peaks, $W \to \infty$. We show in Fig. 6 the sharpness W as a function of $\tilde{T}$. We see that W increases with temperature until it reaches $\tilde{T}_{c1}$ where it drops abruptly.

4. Conclusions

We have presented a numerical study of the effects of noise on the turbulent phase of rf-driven globally coupled JJSA. The breakdown of the law of large numbers is stable at finite temperatures and for weakly disordered arrays. There is both a well defined critical disorder strength δ_c and a critical temperature T_{c1} for the existence of this effect. A remarkable effect is that the sharpness of the broad peaks in the power spectrum *increases* with increasing temperature.

Josephson junction series arrays like the one discussed in this article can be fabricated with the present techniques [9]. In order to observe the breakdown of the law of large numbers, they need to be of high quality, with a spread in the critical currents of no more than 1%. One possible experiment consists in making an underdamped JJSA with a large number of junctions ($N \sim 10^3$–10^5). A measurement of the broad band noise S_0 when cooling the Josephson array

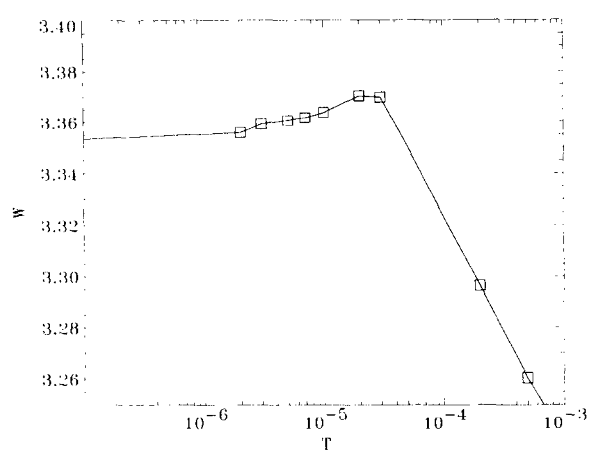

Fig. 6. Measure W of the sharpness of the peaks in the power spectrum as a function of the temperature $\tilde{T}$. $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, $i_{rf} = 0.61$, $i_{dc} = 0.124$, $\sigma = 0.4$ and $N = 16384$.

should show the three temperature regimes described here. First, a decrease of S_0 when decreasing the temperature. Second, a plateau below a temperature T_{c2}. And finally, a sharp increase of S_0 when decreasing T below a critical temperature T_{c1} (for junctions with $I_c = 1\ \mu A$, $T_{c1} \sim 1$ mK, $T_{c2} \sim 0.1$ K). This last regime will be a clear indication of the breakdown of the law of large numbers in JJSA.

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